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Stability and reinforcement analyses of high arch dams by considering deformation effects

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Abstract: The strict definition and logical description of the concept of structure stability and failure are presented. The criterion of structure stability is developed based on plastic complementary energy and its variation. It is presented that the principle of minimum plastic complementary energy is the combination of structure equilibrium, coordination condition of deformation and constitutive relationship. Based on the above arguments, the deformation reinforcement theory is developed. The structure global stability can be described by the relationship between the global degree of safety of structure and the plastic complementary energy. Correspondingly, the new idea is used in the evaluations of global stability, anchorage force of dam-toe, fracture of dam-heel and treatment of faults of high arch dams in China. The results show that the deformation reinforcement theory provides a uniform and practical theoretical framework and a valuable solution for the analysis of global stability, dam-heel cracking, dam-toe anchorage and reinforcement of faults of high arch dams and their foundations.

Key words: deformation reinforcement theory; structure stability; unbalanced force; plastic complementary energy; high arch dams

1 Introduction

During the numerical analysis of geotechnical structures, the structural displacements, stress fields and yielding zones influenced by reinforcements are generally used to evaluate reinforcement effects. But the results of numerous calculations indicate that the reinforcement influence is usually small. Therefore, evaluating the reinforcement effect using this method can result in unreasonable conclusions. In order to resolve this problem, Yang et al. [1–4] developed and applied the deformation reinforcement theory.

High arch dams and their foundations can be regarded as complicated highly hyperstatic structures. They have a great overloading capacity before monolithic failure of dams through local failure phenomena, such as the cracking of dam-heel, the shear compression failure of dam-toe, dislocation of faults. In classical limit analysis, the structure has an ultimate bearing capacity only when a loading path is given. Local failure means that the structure is in the limit state and the external load is the ultimate bearing capacity. So the classical limit analysis cannot analyze the mechanical behavior of structures after local failure.

The deformation reinforcement theory mainly studies the structural behavior or structural failure behavior after the load exceeds the ultimate bearing capacity of structures. The deformation reinforcement theory can be mainly summarized: for the given external loads on structures, there is a region where the unbalanced force leads to the first failure region. In order to maintain the stability of structures, this region needs to be reinforced. The magnitude of reinforcement force is equal to the unbalanced force but its direction is opposite. Minimum plastic complementary energy principle is the foundation of deformation reinforcement theory, and for the given external loads, the structure always has a tendency to approach the lowest possible reinforcement force and the largest possible self-bearing force.

In this paper, the general elastoplastic theory is used to rebuild the theoretical framework of deformation reinforcement theory, and the structure stability theory

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is developed based on the plastic complementary theory. The comparison between the deformation reinforcement theory and the rigid limit equilibrium method is made. Then the global stability of high arch dams is analyzed and compared with that obtained from other methods. Finally, the deformation reinforcement theory is used to analyze the anchorage forces of dam-toes, fractures of dam-heels, and treatment of faults of Baihetan arch dam.

2 Stability definition and instable mechanism of elastoplastic structures

2.1 Associated perfect elastoplastic constitutive relationship

The linear-elastic stress-strain relationship is expressed in tensors and vectors, i.e. $\boldsymbol{\sigma} = \boldsymbol{D} : \boldsymbol{\varepsilon}$ or $\boldsymbol{\varepsilon} = \boldsymbol{C} : \boldsymbol{\sigma}$, where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the second order stress and strain tensors, respectively; $\boldsymbol{D}$ and $\boldsymbol{C}$ are the fourth-order elastic and flexibility tensors, respectively. The incremental elastoplastic constitutive relationship [5] can be written as

$$\mathrm{d}\boldsymbol{\sigma} = \boldsymbol{D} : (\mathrm{d}\boldsymbol{\varepsilon} - \mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}) \tag{1}$$

where $\boldsymbol{\varepsilon}^{\mathrm{p}}$ is the plastic strain tensor. Equation (1) can be further written as

$$\mathrm{d}\boldsymbol{\sigma} = \mathrm{d}\boldsymbol{\sigma}^{\mathrm{e}} - \mathrm{d}\boldsymbol{\sigma}^{\mathrm{p}} \tag{2}$$

where

$$\mathrm{d}\boldsymbol{\sigma}^{\mathrm{e}} = \boldsymbol{D} : \mathrm{d}\boldsymbol{\varepsilon}, \quad \mathrm{d}\boldsymbol{\sigma}^{\mathrm{p}} = \boldsymbol{D} : \mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}} \tag{3}$$

In this paper, the material constitutive behavior is presented in the strain space, in which a strain increment $\mathrm{d}\boldsymbol{\varepsilon}$ is known. As shown in Eq.(1), the main concern of the elastoplastic mechanics is to determine $\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}$. In this paper, the associated flow rule is used, and its yielding condition [6] is

$$f = f(\boldsymbol{\sigma}) \le 0 \tag{4}$$

The consistent condition is

$$\mathrm{d}f = \frac{\partial f}{\partial \boldsymbol{\sigma}} : \mathrm{d}\boldsymbol{\sigma} = 0 \tag{5}$$

The associated normality flow rule is

$$\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}} = \mathrm{d}\lambda \frac{\partial f}{\partial \boldsymbol{\sigma}} \tag{6}$$

According to the consistent condition (Eq.(5)) and the normality flow rule (Eq.(6)), an important relationship can be obtained:

$$\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}} : \mathrm{d}\boldsymbol{\sigma} = 0 \tag{7}$$

Equation (7) is a universal constitutive equation for perfect elastoplastic materials with the associated flow. Constitutive equations of deformation reinforcement theory should be written in an incremental form. Figure 1 shows typical stress adjustments. The initial stress state, $\boldsymbol{\sigma}_0$, is required to be stable, i.e. $f(\boldsymbol{\sigma}_0) \le 0$.

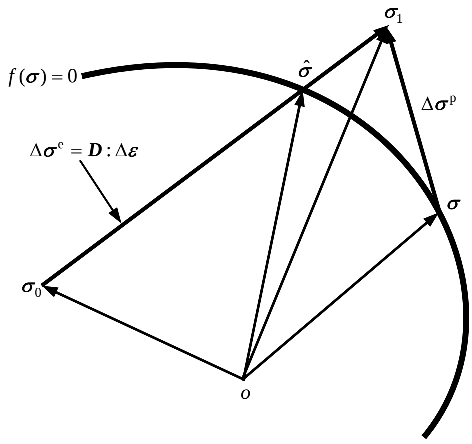


Fig.1 Schematic illustration of elastoplastic stress adjustments.

Given a strain increment $\Delta\boldsymbol{\varepsilon}$ in the initial state, the corresponding stress increment is $\Delta\boldsymbol{\sigma}^{\mathrm{e}} = \boldsymbol{D} : \Delta\boldsymbol{\varepsilon}$, which is assumed to be an elastic loading, $\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_0 + \Delta\boldsymbol{\sigma}^{\mathrm{e}}$. If

$$f(\boldsymbol{\sigma}_1) > 0 \tag{8}$$

the strain increment $\Delta\boldsymbol{\varepsilon}$ results in a plastic loading, otherwise the material response is elastic.

The differential forms of stress and strain increments can be written in incremental forms:

$$\Delta\boldsymbol{\sigma} = \boldsymbol{D} : (\Delta\boldsymbol{\varepsilon} - \Delta\boldsymbol{\varepsilon}^{\mathrm{p}}) \tag{9}$$

$$\Delta\boldsymbol{\varepsilon}^{\mathrm{p}} = \Delta\lambda \frac{\partial f}{\partial \boldsymbol{\sigma}} \tag{10}$$

Equations (9) and (10) can be regarded as the integral forms of Eqs.(1) and (6). According to the integral mean value theorem, Eq.(10) is exactly tenable if $\partial f / \partial \boldsymbol{\sigma}$ is determined by the stress state of a point that is properly chosen according to the path from $\hat{\boldsymbol{\sigma}}$ to $\boldsymbol{\sigma}$. Theoretically, the point exists, but it is difficult to be determined analytically. In this paper, $\partial f / \partial \boldsymbol{\sigma}$ is determined by the final stress state $\boldsymbol{\sigma}$. As described above, the difference between the elastic loading state and the final state is the plastic stress increment, i.e. $\Delta\boldsymbol{\sigma}^{\mathrm{p}} = \boldsymbol{\sigma}_1 - \boldsymbol{\sigma}$. So the plastic stress increment can be expressed as

$$\Delta\boldsymbol{\varepsilon}^{\mathrm{p}} = \boldsymbol{C} : \Delta\boldsymbol{\sigma}^{\mathrm{p}} = \boldsymbol{C} : (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}) \tag{11}$$

Substituting Eq.(11) into Eq.(10), and supposing that $\boldsymbol{\sigma}$ is on the yielding surface, the final stress state can be determined as

$$\boldsymbol{C} : (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}) = \Delta\lambda \left. \frac{\partial f}{\partial \boldsymbol{\sigma}} \right|_{\boldsymbol{\sigma}}, \quad f(\boldsymbol{\sigma}) = 0 \tag{12}$$

If $f(\boldsymbol{\sigma}_1) < 0$, then $\boldsymbol{\sigma} = \boldsymbol{\sigma}_1$. For a loading process, we can get

$$\mathrm{d}\boldsymbol{\sigma} : \Delta\boldsymbol{\varepsilon}^{\mathrm{p}} = 0 \tag{13}$$

2.2 Stability and instability definitions of elastoplastic structures

Given loads and loading paths in elastoplastic structures, classical elastoplastic theory requires that solutions (including displacement fields and stress fields) of structures need to satisfy the equilibrium condition, the deformation compatibility condition and the constitutive relationship. Elastoplastic constitutive relationship includes the yielding condition, which indicates that if a solution of elastoplastic structure exists, the structural stress field satisfies the yielding condition and the structure is stable.

Based on the deformation compatibility of displacement field $\boldsymbol{u}$, for a given residual plastic displacement strain field $\boldsymbol{u}^{\mathrm{p}}$, the corresponding strain field $\boldsymbol{\varepsilon}$ and plastic strain field $\boldsymbol{\varepsilon}^{\mathrm{p}}$, and the elasto- plastic relationship between stress and strain ($\boldsymbol{\sigma} = \boldsymbol{D} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{p}})$), the compatibility stress field $\boldsymbol{S}_k$ can be determined.

Two sets of stress fields, the compatibility stability stress field $\boldsymbol{S}$ and the compatibility equilibrium stress field $\boldsymbol{S}_1$, are both considered. Each element in $\boldsymbol{S}$ is a stress field of $\boldsymbol{S}_k$ satisfying the equilibrium condition, and each element in $\boldsymbol{S}_1$ is a stress field of $\boldsymbol{S}_k$ satisfying the yielding condition. Considering the arbitrary compatibility equilibrium stress field $\boldsymbol{\sigma}_1$ and compatibility stability stress field $\boldsymbol{\sigma}$, their difference is the plastic stress increment field $\Delta\boldsymbol{\sigma}^{\mathrm{p}}$:

$$\Delta\boldsymbol{\sigma}^{\mathrm{p}} = \boldsymbol{\sigma}_1 - \boldsymbol{\sigma} \tag{14}$$

The plastic stress increment field $\Delta\boldsymbol{\sigma}^{\mathrm{p}}$ is associated with the plastic strain increment field $\Delta\boldsymbol{\varepsilon}^{\mathrm{p}}$ in Eq.(11).

Since linear-elastic structures yielding condition does not exist, $\boldsymbol{S} = \boldsymbol{S}_k$. Therefore, $\boldsymbol{S}_1 \cap \boldsymbol{S} = \boldsymbol{S}_1$. It is easy to explain that the real stress field of linear-elastic structures is one element of $\boldsymbol{S}_1$, and the elastic complementary energy is minimum in $\boldsymbol{S}_1$ (the minimum complementary principle of elastic structure [7]).

For elasto-plastic structures, if the intersection of $\boldsymbol{S}$ and $\boldsymbol{S}_1$ is not null, the structure is stable, otherwise the structure is unstable, as shown in Fig.2.

A new Euclidean space about stress field is defined, and one stress field is one point in the space. The distance L between $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}$ can be defined as

$$L^2 = \int_V l^2 \mathrm{d}V = \frac{1}{2}\int_V (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}) : \boldsymbol{C} : (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}) \mathrm{d}V \tag{15}$$

where V is the volume of the structure.

The nearest distance between the stress fields $\boldsymbol{S}_1$

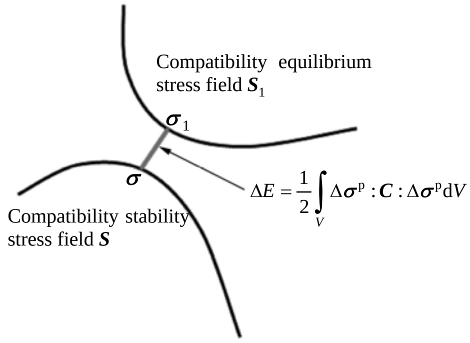


Fig.2 Schematic illustration of elastoplastic solutions in the vector space.

and $\boldsymbol{S}$ is defined as $\hat{L}$:

$$\hat{L} = \min_{\sigma_1,\, \sigma} L \quad (\boldsymbol{\sigma}_1 \in \boldsymbol{S}_1,\ \boldsymbol{\sigma} \in \boldsymbol{S}) \tag{16}$$

Obviously, $\hat{L}$ can be utilized to determine the structure stability: if $\hat{L} = 0$, the structure is stable; if $\hat{L} > 0$, the structure is unstable. The larger the value of $\hat{L}$ is, the more unstable the structure becomes.

The physical meaning of L^2 involves the complementary energy. Therefore, it can be regarded as the plastic complementary energy ΔE:

$$\Delta E = L^2 = \frac{1}{2}\int_V \Delta\boldsymbol{\sigma}^{\mathrm{p}} : \boldsymbol{C} : \Delta\boldsymbol{\sigma}^{\mathrm{p}} \mathrm{d}V =$$

$$\frac{1}{2}\int_V (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}) : \boldsymbol{C} : (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}) \mathrm{d}V \tag{17}$$

Equation (17) shows that ΔE is also the scalar norm of plastic stress increment field $\Delta\boldsymbol{\sigma}^{\mathrm{p}}$. If $\Delta E = 0$, then $\Delta\boldsymbol{\sigma}^{\mathrm{p}}$ is always 0. In this sense, ΔE is also called as the complementary energy norm. The nearest distance can be defined as $\Delta E_{\min} = \hat{L}^2$.

3 Deformation reinforcement theory and its finite element expression

An arbitrary loading can lead to a random strain increment. On the other hand, due to the restriction of ideal elastoplastic yielding surfaces, the stress increment can not be specified arbitrarily because it is unstable if $f(\boldsymbol{\sigma}_1) > 0$. But this problem can be solved by using the non-equilibrium thermodynamics theory, in which non-equilibrium thermodynamics state can be regarded as the constrained equilibrium by introducing internal variables. Thus, the well-established equilibrium thermodynamics theory can be used to deal with the problem of non-equilibrium thermodynamics [8–10]. The driving force of non-equilibrium evolution is the conjugated force. If the non-equilibrium state is imposed by the reverse conjugated force, it will lose the driving force and becomes the constrained equilibrium state.

Since $f(\boldsymbol{\sigma}_1)>0$, it is clear that $\boldsymbol{\sigma}_1$ is unstable or non-equilibrium. If a reinforcement stress increment is imposed to the stress state $\boldsymbol{\sigma}_1$ to make it stable, this stable state is the constrained equilibrium state. Obviously, the reinforcement stress increment is $-\Delta\boldsymbol{\sigma}^{\mathrm{p}}$.

An elastoplastic stable structure is considered here. Its volume is V, the volume force is $\boldsymbol{f}=\{f_i\}$, the stress field is $\boldsymbol{\sigma}=\{\sigma_{ij}\}$, and they satisfy the equilibrium differential equation $\boldsymbol{\sigma}_{ij\,,\,j}+\{f_i\}=\boldsymbol{0}$, yielding condition $f(\boldsymbol{\sigma})\le 0$ and constitutive equation. The structure's displacement field is $\boldsymbol{u}=\{u_i\}$, the strain field is $\boldsymbol{\varepsilon}=\{\varepsilon_{ij}\}$, the geometric equation is $\varepsilon_{ij}=(u_{i\,,\,j}+u_{j\,,\,i})/2$. The total boundary is S_u+S_σ, where S_u is the displacement boundary where $u_i=0$, and S_σ is the stress boundary where $\boldsymbol{T}=\{T_i\}=\{\sigma_{ij}n_j\}$. For an equilibrium stress field $\boldsymbol{\sigma}_1$, the following virtual displacement principle can be obtained:

$$\int_V \delta\varepsilon_{ij}\sigma_{ij}^1 \mathrm{d}V=\int_V \delta u_i f_i \mathrm{d}V+\int_{S_\sigma}\delta u_i T_i \mathrm{d}S \tag{18}$$

According to the randomicity of virtual displacement, the governing equation can be deduced from the virtual displacement principle in Eq.(18):

$$\sum_e \int_{V_e} \boldsymbol{B}^{\mathrm{T}}\boldsymbol{\sigma}_1 \mathrm{d}V=\boldsymbol{F} \tag{19}$$

where e represents the summation to all elements, $\boldsymbol{B}$ is the strain matrix, and $\boldsymbol{F}$ is the equivalent nodal force of external loads:

$$\boldsymbol{F}=\sum_e \int_{V_e} \boldsymbol{N}^{\mathrm{T}}\boldsymbol{f}\mathrm{d}V+\sum_e \int_{S_\sigma^e} \boldsymbol{N}^{\mathrm{T}}\boldsymbol{T}\mathrm{d}S \tag{20}$$

When $\boldsymbol{\sigma}_1=\boldsymbol{\sigma}+\Delta\boldsymbol{\sigma}^{\mathrm{p}}$ is introduced, the virtual displacement principle in Eq.(18) can be expressed as

$$\sum_e \int_{V_e} \boldsymbol{B}^{\mathrm{T}}\boldsymbol{\sigma}\mathrm{d}V=\boldsymbol{F}-\Delta\boldsymbol{Q} \tag{21}$$

where $\Delta\boldsymbol{Q}$ is the equivalent nodal force of the plastic stress field. $\Delta\boldsymbol{\sigma}^{\mathrm{p}}$ is called the unbalanced force in finite element analysis:

$$\Delta\boldsymbol{Q}=\sum_e \int_{V_e} \boldsymbol{B}^{\mathrm{T}}\Delta\boldsymbol{\sigma}^{\mathrm{p}}\mathrm{d}V \tag{22}$$

As shown above, the equivalent nodal force of the reinforcement force is $-\Delta\boldsymbol{Q}$, that is to say, for a specified deformation state, the reinforcement force has the same magnitude but an opposite direction with the unbalanced force. Therefore, at the level of nodal forces, Eq.(21) can be interpreted as structural self-bearing force = external load + reinforcement force or as structural self-bearing force + unbalanced force = external load.

The two expressions are equivalent, but their meaning is different. The former shows the reinforcement force is an external force, while the latter shows the unbalanced force is an internal force.

4 Discussions

4.1 The relationship between reinforcements and displacements

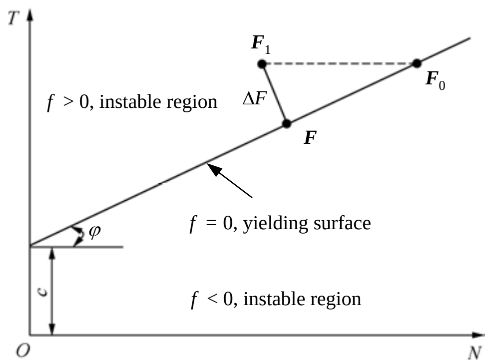
The unstable elastoplastic structure always tends to be the state with the minimum possible reinforcement force and the maximum possible self-bearing force. In the stress field, it is reflected in the continual adjustment of $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}_1$. Since the two stress fields conform to the deformation compatibility, the adjustment process is also a deformation developing process. In the elastoplastic analysis, this process is assumed to be completed instantaneously, but in fact the process takes a certain period of time. If an elasto-viscoplastic model [11] is used, and the external load and loading path are given, the unstable elasto-viscoplastic structure tends to approach the state with the minimum reinforcement force and the maximum self-bearing force by the slow adjustment of deformation. This argument gives a proper explanation to new Austrian tunneling method (NATM). The key of NATM is that the reinforcement force and the reinforcement time are associated with the deformation state. If the displacement needs to be controlled to a certain value, a larger reinforcement force is needed.

The plastic stress field $\Delta\boldsymbol{\sigma}^{\mathrm{p}}$ is determined by the displacement field, which satisfies the deformation compatibility condition. Therefore, the reinforcement force $-\Delta\boldsymbol{\sigma}^{\mathrm{p}}$ is a self-equilibrium stress field. The general reinforcement force is at self-equilibrium indeed, such as the anchorage force. For example, the distribution of anchorage force is complicated along the anchor cable, but the anchor cable is at equilibrium. Therefore, the anchor cable is at self-equilibrium (the gravity of anchor cable is neglected herein). According to the self-equilibrium character and the Saint-Venant's principle, the reinforcement force has little effect on the deformation and the stress of stable structures.

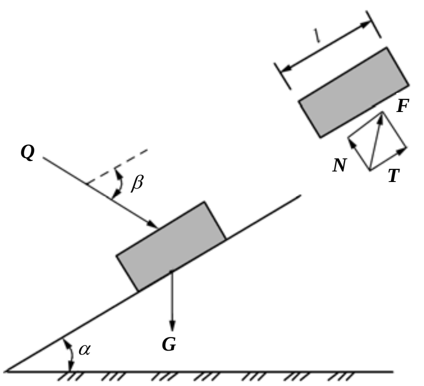
A reinforcement theory needs to determine the reinforced areas and the reinforcement forces of structures. Essentially, the key block theory of Shi [12] is also a kind of reinforcement theory. This theory requires that the key block of blocky system should be reinforced, and the reinforcement force of the key block is determined by the rigid limit equilibrium method. In this sense, areas with unbalanced force can be termed as key areas.

4.2 Comparison with the rigid limit equilibrium method

The rigid limit equilibrium method [13] can be used in the traditional geotechnical reinforcement analysis, as shown in Fig.3. The basic elements of the rigid limit equilibrium and the elastoplastic analysis are the sliding surface and the micro-body (element), respectively. For a sliding surface, there are only two failure modes: expanding and sliding along the surface, and the deformation is not considered in this case. The micro-body allows shear failure in arbitrary directions and all kinds of multi-axis failure modes, which leads to a better adaptability of elstoplastic medium.



(a) Scheme of yielding surface.



(b) Scheme of single sliding block.

Fig.3 Schematic illustration of the reinforcement analysis using the rigid limit equilibrium method.

As shown in Fig.3(b), a single sliding block system representing a slope is considered. The sliding block is subjected to an external load $\boldsymbol{G}$ (such as the deadweight) and a reinforcement force $\boldsymbol{Q}$ (such as the anchorage force). The internal friction angle and the cohesion of sliding surface are φ and c, respectively. The length of the sliding surface is l. The internal force is $\boldsymbol{F}$, and its normal and tangential components are N and T, respectively, i.e. $\boldsymbol{F}=(N,\ T)$. The yielding condition of a sliding surface based on the Mohr-Coulomb criterion is $f=T-N\tan\varphi-cl$, as shown in Fig.3(a).

Firstly, reinforcement forces are not considered. In this case, the internal force of the sliding surface is $\boldsymbol{F}_1$, which can be determined by the equilibrium condition $\boldsymbol{F}_1+\boldsymbol{G}=\boldsymbol{0}$. If $f(\boldsymbol{F}_1)>0$, it indicates that the sliding resistance force of the sliding block is insufficient. As a result, the block needs to be reinforced with the reinforcement force $\boldsymbol{Q}$ to keep stable. Accordingly, the equilibrium condition becomes $\boldsymbol{F}+\boldsymbol{G}+\boldsymbol{Q}=\boldsymbol{0}$, where $\boldsymbol{F}$ is the internal force after reinforcement and satisfies $f(\boldsymbol{F})=0$. According to the equilibrium conditions before and after reinforcement, $\boldsymbol{Q}=\boldsymbol{F}_1-\boldsymbol{F}$. Apparently, the optimal reinforcement force makes the distance between $\boldsymbol{F}$ and $\boldsymbol{F}_1$ shortest with the requirement of $f(\boldsymbol{F})=0$. Actually, this is the extreme case of normality flow rule, in which the normality flow rule is associated with the requirement of the optimal reinforcement forces applied on the sliding surface.

The structural system in the rigid limit equilibrium method is a multi-sliding block system. It is a hyperstatic system, which also needs some extra presuppositions (such as Pan’s maximum and minimum principles [14]) since the relationship between the force and the deformation of the surface has not been specified. Other limit analysis methods, such as DEM, DDA and rigid body surface element method, have introduced the deformation constitutive equations for the sliding surfaces.

5 Global stability evaluation of high arch dams in China

High arch dam is a high-order hyperstatic structure, and its global stability is a deformation stability problem that reflects a failure process. The arch dam failure is characterized with several failure modes. The subordinate failure mode (such as cracking of dam-heel or compression failure of dam-toe) generally happens before the governing failure mode. For the overloading models of failure tests (overload of water bulk density), the basic load is water load P_0, and it is expressed as the degree of safety K. The rupture tests results of arch dams [15] show that the whole process from the elastic state to the failure state of arch dams can be described by using three characteristic global degrees of safety: cracking load K_1, nonlinear deformation load K_2 and failure load K_3, and $K_1 < K_2 < K_3$. These factors bring difficulties and uncertainties to the evaluation of monolithic stability of high arch dams, and a single index cannot be used to evaluate the monolithic degree of safety of high arch dams.

For an overloading process, every load state K is associated with a minimum plastic complementary energy $\Delta E_{\min}$. In this paper, K - $\Delta E_{\min}$ curve is used to evaluate the global stability of arch dams. When $\Delta E_{\min} > 0$, the larger the value of $\Delta E_{\min}$ is, the worse the failure is, which matches with rupture tests. $\Delta E_{\min}$ reflects the deficiency of self-adjusting ability and the whole indices reflect the failure of structure. The coordinate (K, $\Delta E_{\min}$) of an arbitrary point on the curve represents a general ultimate bearing capacity of structure.

Figures 4 and 5 show the responses of plastic complementary energy of foundations and dams of some high arch dams in China, respectively. These responses indicate the magnitudes of global stability of every arch dam.

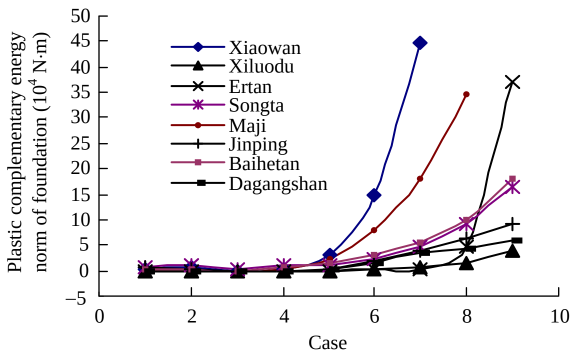


Fig.4 Responses of complementary energy norm of high arch dams foundations in China.

The overloading process is just one of the possible failure paths. For the strength reduction process, similarly, the K_s - $\Delta E_{\min}$ responses can be used to evaluate the global stability, where K_s is the strength reservation coefficient.

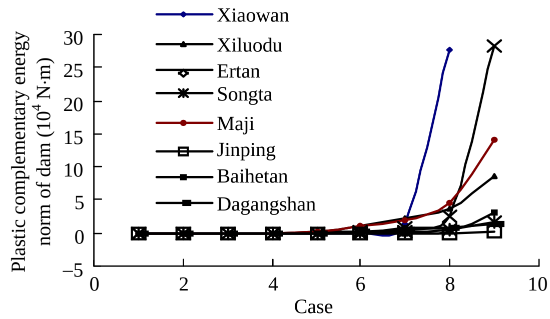


Fig.5 Responses of complementary energy norm of high arch dams in China.

6 Reinforcement analysis of arch dams

The unbalanced force is calculated with the nonlinear finite element method and can be used to analyze the dam-toe anchorage, dam-heel cracking and reinforcement of faults in foundations. The area and reinforcement force are determined by the unbalanced force.

6.1 Anchorage analysis of dam-toe

The dam-toe of high arch dams is a compression-shear concentration zone, and also the first failure zone of arch dam. The stability and reinforcement of dam-toe have an important influence on the stability and safety of high arch dams, and should be designed carefully. The anchorage of dam-toe is an effective reinforcement measure. Lijiaxia arch dam is a high arch dam built on complicated foundations. It adopted the anchorage scheme of dam-toe, and has operated very well since 1997 [16].

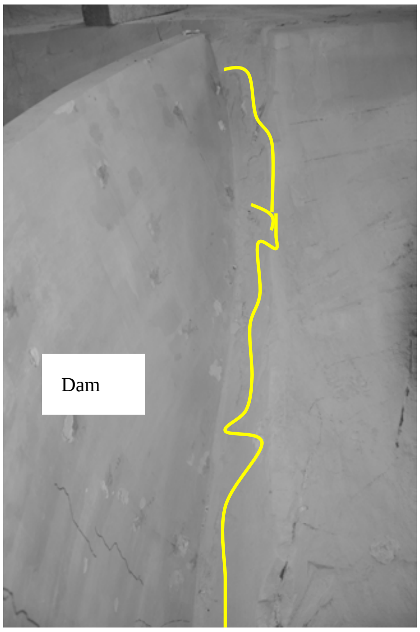
In order to design the anchorage of dam-toe, its unbalanced force needs to be calculated. Table 1 gives the resultant unbalanced force of dam-toe of high arch dams in China. The listed unbalanced forces in Table 1 are all the minimum ones. For a given load case, the structure always approaches the deformation state with a minimum unbalanced force. In this deformation development, the unbalanced force is associated with

Table 1 Unbalanceed forces of dam-toe. (10^4 N)

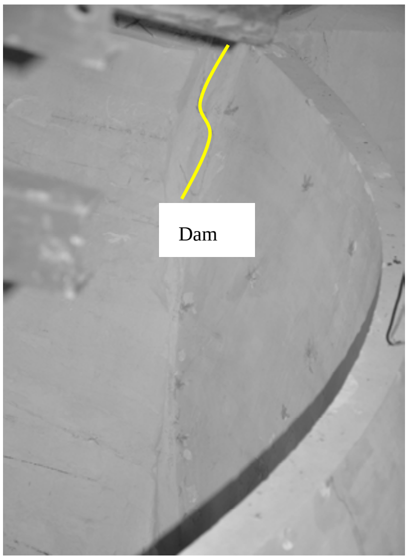
Load case	Xiaowan	Dagangshan	Xiluodu	Ertan	Songta	Maji	Jinping	Baihetan
Normal	19	7 555	27	0	15	32	30	4 615
$1.5P_0$	493	11 799	921	2	6 843	2 459	916	74 407
$2.0P_0$	5 491	25 773	3 596	92	16 348	23 138	5 228	149 290
$2.5P_0$	30 844	38 291	9 832	806	31 081	43 680	17 028	237 704
$3.0P_0$	96 410	52 128	23 546	3 742	50 387	66 476	28 267	336 131
$3.5P_0$	220 573	69 612	44 692	13 810	72 315	97 767	38 830	454 739
$4.0P_0$	415 063	87 337	72 382	40 507	97 111	136 665	52 363	599 163

the deformation. That is, different reinforcement forces are needed for different nonlinear deformations.

For Baihetan arch dam, the unbalanced force of left bank (44.5×10^8 N) is larger than that of the right bank (0.68×10^8 N). The rupture test of Baihetan arch dam [17] shows that the damage of the left bank is more serious than that of the right bank, as shown in Fig.6.



(a) Left bank.



(b) Right bank.

Fig.6 Failure of Baihetan arch dam.

6.2 Fracture analysis of dam-heel

Cracking of dam-heel is an important issue in arch dam designs. It is very difficult to conduct a fully 3D cracking analysis using fracture mechanics for high arch dams, and the cracking criterion is mainly stress-based. As mentioned above, high arch dams are high-order hyperstatic structures and have strong self-adjusting capabilities against failures, and the deficiency of self-adjusting capability is associated with the unbalanced forces. Therefore, the cracking modes of dam-heels can be determined through the unbalanced forces analysis.

Figure 7 demonstrates the unbalanced force vectors of the upstream dam and the foundation of an arch dam, where the unbalanced force in the area is prone to cause cracking. Table 2 summarizes the unbalanced forces of the dam-heel. The fracture analysis can be performed using the data in Table 2.

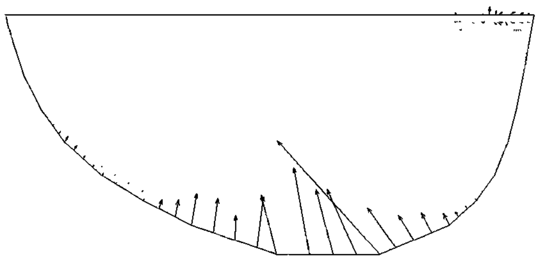


Fig.7 Unbalanced force vectors of the dam body for an arch dam.

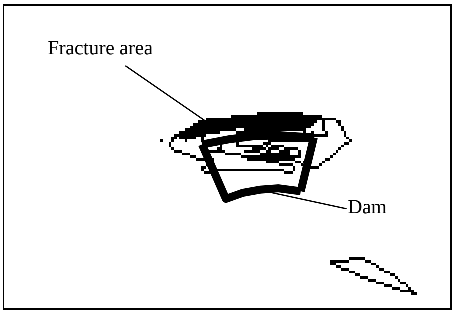
Table 2 Unbalanced forces of dam-heel. (10^4 N)

Load case	Left				Right			
	F_X	F_Y	F_Z	F	F_X	F_Y	F_Z	F
Normal	0	0	0	1	0	0	0	0
$1.5P_0$	301	5 111	1 666	5 384	1 114	10 745	4 153	11 573
$2.0P_0$	981	8 687	4 249	9 720	3 685	18 808	7 944	20 747
$2.5P_0$	2 604	12 760	9 312	16 010	6 905	25 601	12 517	29 322

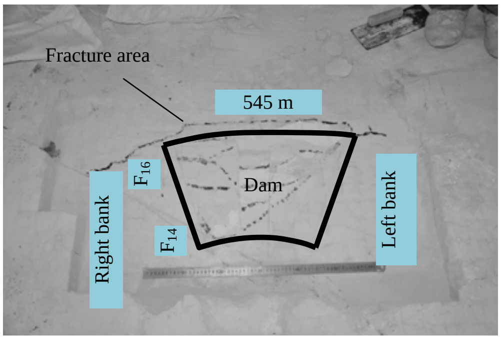
Comparisons of the unbalanced force and the fracture area in the rupture test are shown in Fig.8. The unbalanced force appears near the dam-heel, where the fracture area is located.

6.3 Fault treatments in foundations

Faults and weak zones in foundations of abutments often control the stability of arch dam. Generally, there are many faults in the abutments of arch dams. It is important to identify the critical faults. On the basis of elastoplastic analyses, the distributions, directions and quantities of the unbalanced force of faults and weak zones can be used to guide reinforcement designs.



(a) The distribution of unbalanced forces.



(b) Fracture area of rupture test.

Fig.8 Comparisons between the unbalanced force and the fracture area in rupture test.

Table 3 shows the unbalanced forces of faults in the foundations. According to Table 3, the critical fault can be determined. For the critical fault, the rein force-ment can be designed based on the distributions and directions of the unbalanced forces, as shown in Fig.9.

Table 3 Unbalanced forces of faults. (10^4 N)

Load case	C_3	$C_{3\text{-}1}$	C_4	LS_{337}	LS_{331}	LS_{3318}
Normal	520	90	10	30	10	4 980
$1.5P_0$	18 380	26 980	3 840	50	3 130	64 510
$2.0P_0$	31 230	47 130	11 670	140	21 390	132 440
$2.5P_0$	39 640	65 080	26 570	230	65 130	219 140
$3.0P_0$	48 730	86 480	50 530	440	135 870	342 480
$3.5P_0$	61 180	123 230	88 150	1 230	263 270	522 300
Load case	F_{14}	F_{16}	F_{17}	F_{18}	f_{108}	$C_{3\text{-}1}$
Normal	430	10	700	22 630	90	90
$1.5P_0$	6 450	5 310	3 810	49 210	2 110	26 980
$2.0P_0$	15 700	11 060	10 150	92 950	7 990	47 130
$2.5P_0$	32 210	22 760	28 470	180 300	24 290	65 080
$3.0P_0$	68 980	39 360	67 000	371 780	64 230	86 480
$3.5P_0$	140 380	63 650	121 060	728 690	125 540	123 230

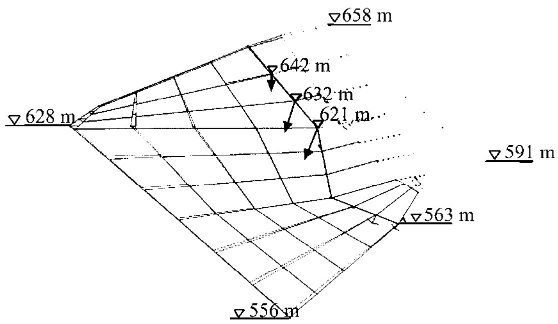


Fig.9 Unbalanced force distribution of fault LS_{3318}.

7 Conclusions

Deformation reinforcement theory is the advancement of traditional structural limit theory based on the principle of minimum plastic complementary energy. It can be used to study the structural failure when the external loads exceed the ultimate bearing capacity, and determine the structural reinforcement forces. It reflects the characteristics of deformation stability of arch dams. The principle of minimum plastic complementary energy consists of the equilibrium condition, the deformation compati-bility condition and the constitutive relationship. The plastic complementary energy determines the develop-ment of structural instability, and is also a guide to determine reinforcement forces. The plastic complementary and its variation can be used to evaluate the structural stability. The structural global stability can be represented by structural global degrees of safety and the plastic complementary energy.

Deformation reinforcement theory provides a uniform theoretical framework and a basis for monolithic stability evaluation, determinations of anchorage of dam-toes, reinforcements of faults of high arch dams and so on.

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